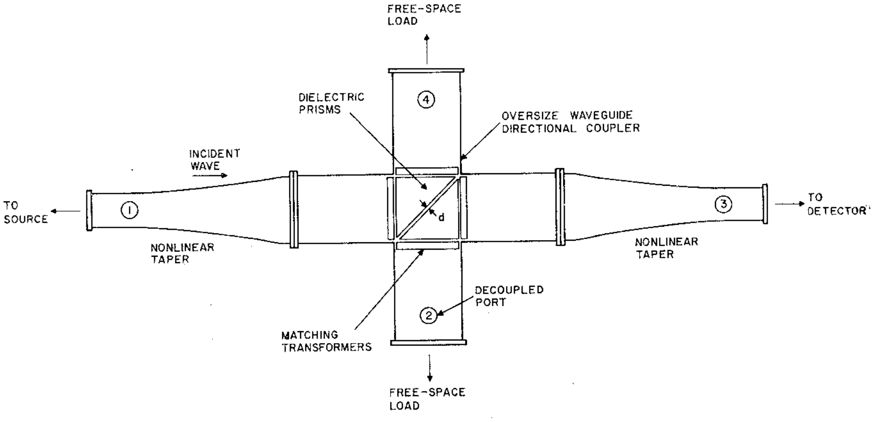


Fig. 1—Oversize waveguide directional coupler schematic.

signed for the 10-cm range; a millimeter wavelength version can be designed by direct scaling.

Fig. 1 shows the construction of the coupler including nonlinear tapers[4] used to launch a pure TE_{10} mode in the oversize guide. Fig. 2 is a picture of the device showing the dielectric coupling elements. The oversize guide had *a* and *b* dimensions of 8.51 and 5.82 inches respectively. This guide is about 3 times oversize for standard *S*-band. The coupler consists of a pair of dielectric prisms ($\epsilon_r = 2.55$) separated by an air space of thickness *d*. Quarter-wavelength dielectric slabs ($\epsilon_r = 1.6$) were used to match the prisms to the four waveguide ports. The theory follows an analysis for a pair of optical prisms.[1] Power incident at port 1 is coupled to ports 3 and 4, where the relative division is a function of the spacing between the two prisms. Port 2 is isolated from port 1. The theoretical expressions for power coupling to ports 4 and 3 (assuming a lossless dielectric) are

$$\frac{P_4}{P_1} = \frac{2.4 \sinh^2 (3.29\, d/\lambda)}{\sinh^2 (3.29\, d/\lambda) + 1.40 \cosh^2 (3.29\, d/\lambda)}$$

and

$$\frac{P_3}{P_1} = 1 - \frac{P_4}{P_1}$$

where

P_1 = power available at port 1,
P_4 = power coupled to port 4,
P_3 = power coupled to port 3,
λ = free-space wavelength.

We have constructed a 3-db directional coupler wherein the coupling to ports 3 and 4 is equal within 0.3 db and the insertion loss is about 0.8 db over a 15-per cent band centered about 2.9 Gc. The results are shown in Fig. 3. This loss was mainly due to a 0.20-db loss in the tapers, a 0.30-db dielectric loss, and wall losses. The power coupled to port 2 is 21 to 26 db down from the incident level over the band. The input SWR is about 1.3 over the band. Additional experimentation at different *d*'s agree with the theory.

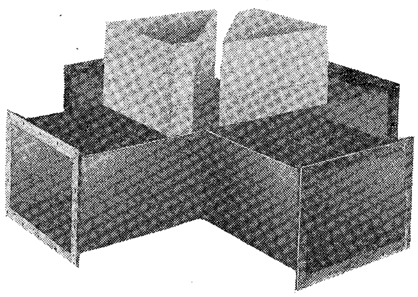


Fig. 2—Oversize waveguide directional coupler showing coupling prisms.

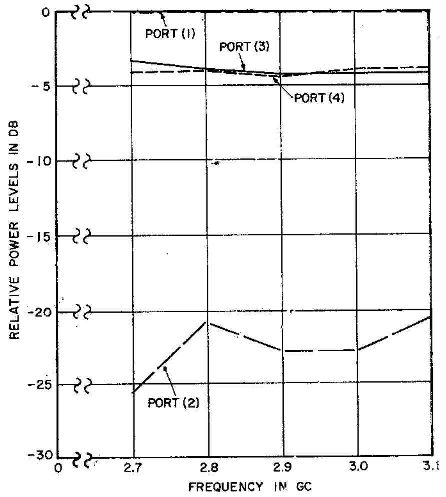


Fig. 3—Frequency response of over-size directional coupler.

Further work is in progress to scale this device to the millimeter region and to reduce the wall loss due to higher mode conversion. A device of this type can be used as an adjustable directional coupler and variable attenuator by mechanically varying *d*. By setting *d* for an equal power split, it becomes a 90° hybrid and thus can be used to form other components such as variable phase shifters and mixers.

H. J. Hindin
J. J. Taub
Airborne Instruments Lab.
Deer Park, L. I., N. Y.

---

[4] H. G. Unger, "Circular waveguide taper of improved design," *Bell Sys. Tech. J.*, vol. 37, pp. 899–912; July, 1958.

Resonances in a Cylindrical Plasma Column*

A plasma column irradiated by an electromagnetic wave which has its electric vector and direction of propagation perpendicular to the axis of the tube exhibits Tonks-Dattner[1,2] resonances in its absorption pattern when the electron density in the column is varied. The pattern consists of a main resonance and a number of less pronounced subsidiary resonances at currents corresponding to electron densities lower than that of the main peak. The subsidiary resonances grow progressively smaller as the electron density decreases.

Herlofson[3] predicted a single resonance for a cylindrical plasma irradiated in this fashion. He treated the problem by solving the wave equation in cylindrical coordinates subject to appropriate boundary conditions, and found only one frequency for maximum scattering from the column. No resonances at all were predicted for the parallel mode of excitation in which the *E* vector is parallel to the axis of the column. So far no completely satisfactory theory has been put forward to account for the subsidiary resonances.[4,5]

In the course of experiments attempting to elucidate this phenomenon, our observations have shown that these resonances may also be seen when the electric field is parallel to the axis of the plasma column. Furthermore, if the plasma column is placed with its axis parallel to the direction of propagation in a waveguide or transmission line and the length of the column is of the order of a wavelength and can be no longer considered as a localized discontinuity, the Tonks-Dattner resonances are still clearly shown. The following experimental results illustrate these points:

Fig. 1(a) shows the normal Dattner experiment with the plasma column across a waveguide propagating a TE_{01} mode, together with the resulting absorption pattern when the current through the plasma tube is varied.

Fig. 1(b) shows the waveguide rotated through 90° with respect to the plasma column as well as the absorption pattern which again shows the Tonks-Dattner resonances.

Fig. 2(a) shows a plasma column placed across a parallel plate transmission line and a photograph of the transmission down the line as the current in the tube is varied. Above a certain current the transmission is cut off at current levels; below this we have the characteristic Tonks-Dattner resonances.

---

* Received April 23, 1962. This work was supported in part by a grant from the National Science Foundation.

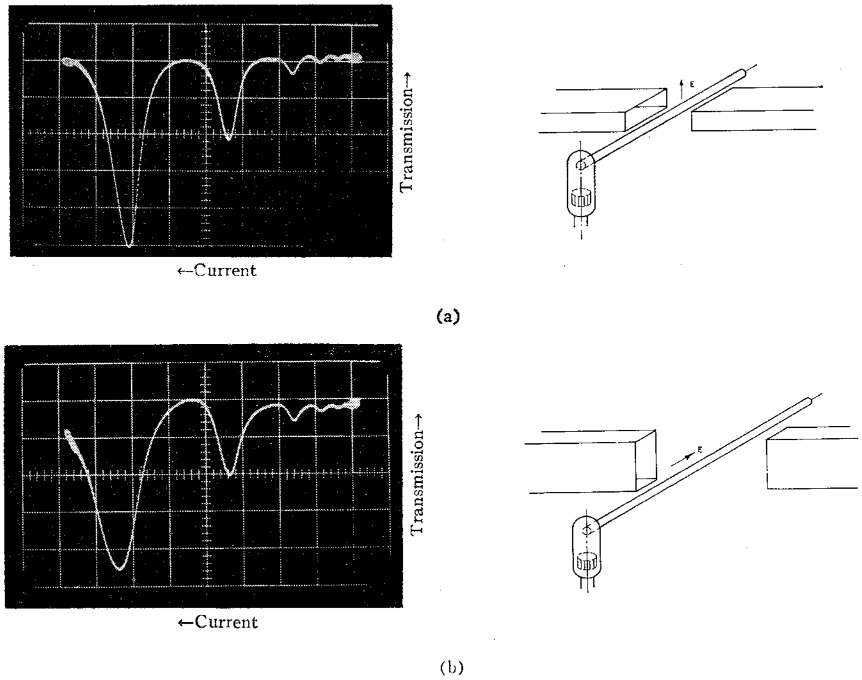
[1] L. Tonks, "The high frequency behaviour of a plasma," *Phys. Rev.*, vol. 37, pp. 1458–1483; June, 1931.

[2] A. Dattner, "The plasma resonator," *Ericcson Technics* (Stockholm), vol. 13, no. 2, pp. 310–350; 1957.

[3] N. Herlofson, "Plasma resonance in ionospheric irregularities," *Arkiv Fysik*, vol. 3, no. 15, pp. 247–297; 1951.

[4] W. D. Hershberger, "Absorption and reflection spectrum of a plasma," *J. Appl. Phys.*, vol. 31, pp. 417–422; February, 1960.

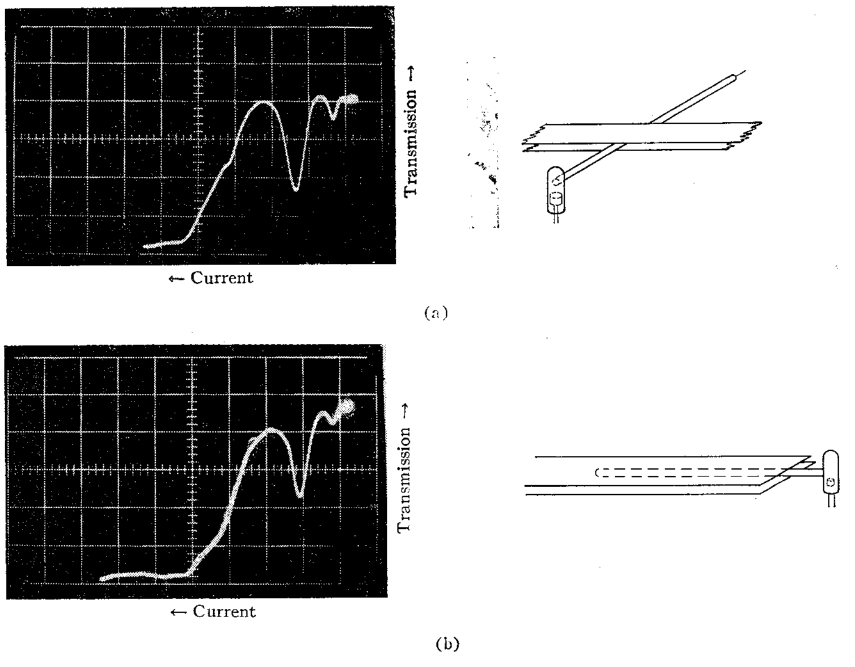
[5] R. W. Gould, "Experiments on plasma oscillations," *Proc. Conf. on Plasma Oscillations*, Spencer, Ind., pp. 167–206; June, 1959.



(a)

(b)

Fig. 1—(a) E field perpendicular to column. (b) E field parallel to column.



(a)

(b)

Fig. 2—(a) Plasma tube across transmission line. (b) Plasma tube along transmission line.

If now the plasma tube is placed at the center of the transmission line with its axis parallel to the axis of the line as shown in Fig. 2(b) the resulting output from the line again exhibits the Tonks-Dattner resonances. The tube in this case was approximately 1.2 wavelengths long.

The above experiments show that resonances can be excited when the electric vector of the incident EM wave is parallel to the axis of the plasma column and also when the plasma column is placed so that it may not be considered as a localized discontinuity. Since these resonances are not predicted by any theory so far put forward we feel there is a need for variation of the Dattner type experiment and for the performance of new and different experiments so that the conditions under which these oscillations are excited can be more completely determined.

J. Willis
I. Petroff
College of Engineering
University of California
Los Angeles, Calif.

Proposed Experiment for Eliciting Multiple Resonances from the Ionosphere*

In the theory of Herlofson[1] only one resonance is predicted for a cylindrical plasma column irradiated by an electromagnetic wave having both its direction of propagation and electric field E perpendicular to the axis of the column, a mode which he designates as sagittal. Herlofson treats the problem by solving the wave equation in cylindrical coordinates and then imposing boundary conditions to find the frequency or frequencies for maximum scattering from the column. In his treatment, the modes which involve Bessel functions of order higher than unity have the same resonant frequency as that for the dipolar mode for which the order of the Bessel function is unity. No resonances at all are predicted for the parallel mode of excitation in which E is parallel to the axis of the column. These predictions are contrary to the experimental observations of Dattner[2] and others[3] for the saggital mode and also contrary to the observations reported by Willis and Petroff[4] in which a spectrum of resonances is found for the parallel mode. Experiments by Boley[5] have shown that the sagittal scattering for the higher order resonances is that appropriate for a dipole, that is, his experiments show that the field about the column for the higher-order modes is not quadrupolar or sextupolar.

These multiple resonance experiments under a variety of experimental arrangements suggest that the mode spectrum may be an intrinsic property of a plasma, perhaps of an extended plasma such as the ionosphere. The higher-order resonances occur when the electron density is lower than that required for the principal resonance. The frequencies for the various resonances, f_n (they may also be expressed in terms of electron densities), are given reasonably well by the expression

$$(2n)/(2n+1) = \left[1 - \left(\frac{f_0}{f_n}\right)^2\right]^{1/2} \qquad (1)$$

where $n=0$ for the principal resonance and $n=0, 1, 2, 3, \cdots, n$ for the series. f_n may be expressed in terms of f_0 and, for the various values of n we have

$$f_n/f_0 = 1.0,\ 1.34,\ 1.67,\ \text{etc.} \qquad (2)$$

An experimental test of this hypothesis would be afforded by taking ionospheric soundings employing a set of discrete frequencies and noting whether reflections from the ionosphere are obtained, at a fixed height, not at one frequency only, but

---

* Received May 4, 1962.

[1] N. Herlofson, "Plasma resonance in ionospheric irregularities," *Arkiv. f. Physik.*, vol. 3, pp. 247–297; 1951.

[2] A. Dattner, "The plasma resonator," *Ericsson Technics* (Stockholm), vol. 13, pp. 310–350; 1957.

[3] W. D. Hershberger, "Absorption and reflection spectrum of a plasma," *J. Appl. Phys.*, vol. 31, pp. 417–422; February, 1960.

[4] J. Willis and I. Petroff, "Resonances in a cylindrical plasma column," IRE Trans. on Microwave Theory and Techniques, September, 1962.

[5] F. I. Boley, "Scattering of microwave radiation by a plasma column," *Nature*, vol. 182, pp. 790–791; September 20, 1958.